Fighting Collaborative Tax Evasion

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Abstract

We build a model of collaborative tax evasion where a buyer negotiates a price discount with a seller in exchange for not asking the receipt and paying in cash, allowing tax evasion. Sellers and buyers are heterogeneous with respect to their tax morale and to their cost of managing payment instruments different from cash. We study how a tax rebate for the buyer and a tax on cash withdrawals (TCW) affect tax evasion and government revenue. An appropriate mix of these two instruments can reduce tax evasion and, at the same time, increase revenue. The TCW is effective only if sufficiently high, and it must be higher the higher the evasion rate in the country and the larger the mass of individuals using cash as a paying instrument. We discuss the implementation problems of the TCW and we suggest how to overcome them.

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**Keywords:** collaborative tax evasion; tax on cash

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1 Introduction

The first economic model of tax evasion by Allingham and Sandmo (1972) explains evasion as a result of a cost benefit analysis by perfectly rational sellers of goods or services. They choose to evade if the expected cost of the sanction, given the auditing probability, is lower than the tax payments. A great deal of economic literature followed their pioneering work (Sirinvasan 1973, Yitzhaki 1974, Baldry 1979, Marrelli 1984, Reinganum and Wilde 1985, Usher 1986, Marrelli and Martina 1988, Andreoni 1992, among others), adding many elements to their baseline framework.

What is in general missing in this literature, besides few exceptions, is the role of the buyer: asking for a receipt of the transaction makes tax evasion more difficult, while paying cash without asking for a receipt, leaving no proof of the transaction, facilitates it. Since buyers have the power to ease or impede tax evasion, it is plausible that some sellers will try to induce a cooperative behavior from them, for instance offering a price discount. When the two parties reach an agreement, a form of “collaborative tax evasion” takes place.

Our goal is to identify the optimal policies to curb this collaborative tax evasion while, at the same time, raising additional tax revenue. Besides the tax rate, we consider two policy instruments: a tax deduction for the buyers that keep the receipts of the transactions and a tax on cash withdrawals (TCW henceforth).

The tax deduction (or tax credit) is a standard instrument, implemented in many tax codes around the world and sometimes very creatively: in Taiwan, China, Puerto Rico and in the city of Sao Paulo, for instance, the receipt can be used to claim lottery tickets (Marchese 2009, Fabbri 2013). The purpose of tax deductions is to reduce evasion by rewarding honest taxpayers rather than punishing dishonest ones and many studies suggested that this strategy can be effective1 (Among others, Alm, McClelland, and Schulze 1992 and Berhan and Jerkins 2005.) The TCW, conversely, is a rather new instrument. Its purpose is obviously to curb the incentives of the buyer to cooperate with the seller2: if evasion can be perpetrated only in case of cash transactions, then making the use of cash more costly should induce less cooperation. We are aware of only

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1Frey (1997) also points out that implementing punishing schemes might actually crowd out the intrinsic motivation to pay taxes and can also result in more tax evasion. This result is consistent with experimental evidence that suggests a drop in cooperation when fines for deviations are introduced (Scholtz and Lubell 2001).

2There exist also some literature against the effectiveness of policy instruments that reduce the incentives to cooperate on tax evasion, like Santoro (2006).
two countries that implemented this tax, Pakistan in 2001 and India from 2005 to 2009 (the so called Banking Cash Transaction Tax or BCCT). In both cases, however, the official reason for the introduction of the tax was not a reduction of tax evasion nor the collection of additional tax revenue. Instead the tax was supposed to provide more information for the tax enforcing authorities to better guide the audits\(^3\). Here we provide a throughout analysis of the effects of the TCW and of its interactions with the tax deductions. Differently from India and Pakistan, however, we focus on the role of these instruments as tools to both reduce tax evasion and increase government revenue.

We build a model where the price taking sellers (small vendors, family businesses or independent professionals) choose how much of their aggregate tax liability to report and where the customers decide whether to facilitate tax evasion, accepting a price discount for not asking a receipt of the transaction. If the seller chooses to evade, he enters in a bargaining round with the customer. If the parties reach a deal, then the customer receives a discount and the transaction is completed without receipt. In this case the transaction must be completed in cash, since credit or debit cards, bank transfers, checks and other payment instruments leave a paper trial. If there is no deal, there is no discount for the customer and the seller issues the receipt. In this case the buyer is not constrained to use cash. Sellers and buyers are heterogeneous with respect to their honesty, or tax morale, and with respect to their cost of managing payment instruments different from cash. In the baseline model, we assume that the characteristics of buyers and sellers is public information and we use the Nash bargaining solution. Prior to the evasion decision by the seller and before the bargaining game, a legislator, whose objectives are to fight tax evasion and to maximize revenue, commits to a tax rate, to a tax rebate and to a TCW rate.

Although the model is highly stylized along many dimensions, it is still not possible to completely solve it analytically. Therefore we proceed as follows. First, we derive analytical results for the effects of the policy instruments on tax evasion, for which we have an explicit solution. For the effects of the policy instruments on the government revenue and for the optimal policy we resort to a numerical solution. We calibrate the model to a fictitious “prototype economy”, arbitrarily choosing both the value of the parameters and the calibration targets. We choose this

\(^3\)Indeed, the tax in India was abolished in 2009 exactly claiming its scarce contribution once more sophisticated information collection technologies were implemented.
approach for two reasons: first because we want to highlight the principles that should guide the anti-evasion policy in general, for a large set of countries, instead of focusing attention on a single country. In fact we also make an effort to study the robustness of all the results to a wide range of alternative parameterizations and calibrations. Second, because it is difficult to identify the exact value of some of the key economic variables that we need in order to credibly calibrate the model to isolate a policy for a specific country.

We show that a small tax deduction is effective at both reducing tax evasion and increasing government revenue and that the deduction rate must be higher the higher the rate of tax evasion in the country and the higher the statutory tax rate. We also show that the TCW can actually increase evasion in economies or sectors of the economy where the use of cash is high. The reason is that, for individuals with high costs of using alternative paying instruments, taxing cash raises the threat value in the bargaining process, inducing more rather than less evasion. Obviously the higher the TCW, the smaller the percentage of individuals that will use cash. We show that the first effect prevails for a small rate of the TCW, while the second for high rates. Therefore we conclude that taxing cash is effective at reducing evasion and increasing revenue only if the rate is high enough, and the TCW rate must be higher the larger the mass of individuals with high costs of using alternative paying instruments.

We then isolate the optimal policies to maximize the government revenue while, at the same time, keeping evasion below a low threshold value. Overall, the main result of the analysis is that, with an appropriate mix of cash taxes and tax rebates, it is possible to curb tax evasion and, at the same time, to raise a substantial amount of additional tax revenue. We also provide an example of the optimal policy for a real world country, Italy.

The main problem with our optimal policy, however, is that the TCW is not easy to implement. On the one hand, the TCW can foster the emergence of a parallel cash economy (Gordon 1995, Morse et al. 2005), significantly reducing its incidence: firms and consumers can use whatever cash they have for the transactions, bypassing the banking system. Besides the reduced incidence of the tax, this will have important side effects for the entire economy, as the financial intermediation improves the functioning of the payment system, fostering investments and growth. In addition, there is the possibility of a bank run at the moment of the announcement of the tax and before its introduction. Moreover, since banks typically charge a fee for credit card use to both sellers and
buyers, this can translate in a loss of profits for the sellers and in a loss of purchasing power for the buyers. Finally, the TCW should be implemented in all the countries that issue the same currency and, to avoid arbitrage, the rate should be equal or, at least, not very different. We propose a throughout discussion of these issues, suggesting how to overcome the implementation problems.

While the symmetric information assumption between seller and buyer might be sensible for repeated interactions, for spot transactions it is more reasonable to assume asymmetric information. We explore also this possibility by modeling the bargaining process as a double auction à la Chatterje and Samuelson (1983). Overall, the results are very similar.

The rest of the paper is organized as follows. Section 2 briefly summarizes the related economic literature. Section 3 describes the model and the analytical results. Section 4.1 illustrates the model calibration for the prototype economy. Section 4.2 reports the comparative static results for the prototype economy and its robustness. Section 4.3 discusses the optimal policy for the prototype economy and its robustness. In Section 5 we discuss the implementation problems of the TCW and propose how to overcome them. Section 6 summarizes the results obtained under the assumption of asymmetric information. Section 7 offers some concluding remarks. Appendix A specializes the results of Section 4.3 for Italy and Appendix B provides the analytical solution of a simplified version of the model under complete and incomplete information.

2 Related literature

The paper follows quite abundant economic literature on tax evasion. We do not include a complete review of the works on tax evasion since there already are very good and exhausting ones in the literature (Andreoni, Erard and Feinstein 1998, Slemrod and Yitzhaki 2002, Cowell 2004, Marchese 2004, Sandmo 2005, Slemrod 2007 and Franzoni 2008). We limit ourselves to a brief account of the few works that specifically tackled collaborative tax evasion.

The first work on collaborative tax evasion is Gordon (1990), that suggests that the under-the-counter cash sales at a discount price, on which the seller evades taxes, serve also as a means of price discrimination. Namely the receipt of the transactions is the basis on which all the after sale services are based, including the possibility of returning a defective item and the possibility of suing the seller. Customers who are not interested in this insurance, for various reasons, are
willing to pay a lower price. The second work is Boadway, Marceau and Mongrain (2002), that model evasion as a collusion between two taxpayers, the buyer and the seller. They assume that cooperative tax evasion efforts can reduce the detection probability more than individual efforts, which gives an incentive to cooperation, and analyze the possibility of punishment schemes for the non cooperating parties. They show that tax evasion might increase after an increase in sanctions because it leads to a larger reduction in the deviation payoff. The third work is Chang and Lai (2004), that models collaborative tax evasion as a bargaining game between a seller and a buyer. The authors are interested in the role of social norms (Akerlof 1980) in shaping the incentives of the agents: evading taxes induces psychological costs, associated to the feelings of guilt, shame etc., but these costs are higher if there is a higher social sanction of evasion, that is, the lower the tax evasion rate. They also show that, contrary to the standard tax evasion model, if the economy is in a bad equilibrium with widespread evasion, a more severe fine or enforcement can actually induce more tax evasion: a higher expected fine raises the gains from trade and, given the low social sanction, evasion increases.

Differently from these previous works on collaborative tax evasion, in this paper we take a normative perspective and we focus on different policy instruments than enforcement and fines. The paper is also related to the literature on the inflation tax (Friedman 1969; Phelps 1973; Chamley 1985; Woodford 1990, among others). Like the inflation tax, the TCW reduces the purchasing power of the consumers. Unlike the inflation tax, however, it is selective, in the sense that it reduces the purchasing power only if the goods or services are paid in cash. Therefore its incidence is lower the bigger the possibility of using the electronic currencies and the lower the cost associated to credit and debit card use for the customers. In addition, the introduction of the TCW, by reducing the number of transactions in cash, will most likely reduce the seigniorage revenue. Although this source of revenue is only of limited importance for low inflation, advanced, economies, it can be important for developing economies which are not able to manage a tax collection system or that have high rates of irregular activity. Nicolin (1998) and Koreshkova (2006) already discussed the role of the inflation tax as a way to raise revenue from tax evaders and from the businesses that operate in the underground sector. With respect to their work, in this paper we take a different perspective and study how to increase the tax revenue by reducing evasion, rather then how to extract revenue from the evaders.
3 The Model

We propose a game-theoretic approach to tax compliance where the buyers, the sellers and the government interact strategically. The economy is composed by a unit measure of price taking, risk neutral, sellers, a unit measure of risk neutral customers and by the government. We model a single transaction between a seller and a buyer about a given good or service. The government implements a tax and enforces the law with random audits resulting in a fine for those who are caught evading. The seller can evade taxes by not issuing the receipt, failing to report the transactions to the tax authority. We assume that, in case there is a receipt, it is impossible for the seller to evade taxes. We also assume that the buyer has the right to ask for a receipt and that the seller cannot refuse to provide one. In this setting, which is indeed similar to what happens in many real world situations (doctors, contractors, plumbers, shops etc.), a negotiation between the seller and the buyer is likely: the seller might offer a price discount to the buyer in exchange for not issuing the receipt, allowing tax evasion. This situation has been called, in the economic literature, collaborative tax evasion to stress that the buyer’s cooperation is essential. If the buyer cooperates, we assume that the good is paid in cash, since electronic payments leave a trace that impairs any evasion attempt.

The sellers are heterogeneous with respect to their tax morale: honest sellers will always issue the receipt, while less honest sellers will always bargain with the buyers. The buyers are heterogeneous along two dimensions. The first is tax morale, as for the sellers. Honest buyers will always ask for a receipt, preventing the buyers from evading, while less honest buyers will always bargain. We consider tax morale in our model since, as noted by Gordon (1989), Andreoni, Erard and Feinstein (1998) and Feld and Frey (2002), it is almost impossible to account for the observed compliance rate just with expected penalties and risk aversion.\footnote{According to Gordon (1990), there is also another reason, beyond honesty, for which the buyers need a receipt: to have a formal guarantee on the product, so that, for instance, they can return a defective item. Following this argument, more risk averse consumer should ask for a receipt more often. In what follows, we will ignore this feature, mainly because, as narrative evidence suggests, the sellers typically offer the same kind of customer service even when the fail to provide the receipt (in fact this is part of the argument they use to convince the buyers to go without receipt).} The second heterogeneity concerns the cost associated with managing payment instruments different from cash. Eletronic payments have greater adoption rates among young adults who are more open to new products (Humphrey...
et al. 2001). Indeed, some individuals, like the elderly or the less financially educated, might find it cumbersome to use a credit card, while others might find it easy. Moreover, some consumers (but not all) are uncomfortable with the idea that their purchases will be tracked and are ready to pay a price for an anonymous method of paying. Garcia Swartz et al. (2006) estimates the privacy loss from methods of paying that display personal information (check or plastic card) using loyalty card discounts.

The government announces and commits to a policy \( P = \{t, \tau, \vartheta\} \). A policy is made up of a tax rate \( t \) and of two additional instruments, a tax on cash withdrawals or TCW \( \vartheta \) and a tax deduction for the buyers that keep the receipt of the transactions \( \tau \), both of which are designed to reduce the incentive of the buyer to cooperate. We assume that the government objective is to maximize the revenue from taxation minus the cost, imposed on honest people as an effect of the TCW, of managing payment instruments different from cash.

After observing the policy \( P \), one buyer and one seller are randomly matched and play the following game. First, seller and buyer choose the amount to evade (possibly zero) and the mean of payment (cash or electronic payment), respectively; then a negotiation between buyer and seller over the price discount takes place and we utilize the Nash bargaining solution to solve it.

In the next sections, we describe in greater detail the case of bargaining under complete information. This setting entails an analytical solution for the equilibrium tax evasion rate and for the government revenue that allows us to highlight the main trade offs of the Government policies.

### 3.1 Sellers and Buyers

The utility of the seller in case of tax evasion \((e)\), which requires cooperation from the buyer, is the following:

\[
\nu^e_s = (1 - \pi) \left[ p(1 - t) + te - d - v \right] + \pi \left[ p(1 - t) - d - fte - v \right] = p(1 - t) + te \left[ 1 - \pi \left( 1 + f \right) \right] - d - v
\]

where \( p \) is the price of the good (taken as given by the seller), \( t \) is the tax rate, \( d \) is the discount bargained with the buyer, \( \pi \) is the audit probability, \( f \) is the fine and \( v \) is the individual cost of tax
evasion, which reflects differences in honesty between sellers. In case of no audit, with probability $1 - \pi$, the seller earns the evaded amount $te$ minus the discount. In case of audit, the seller is forced to pay the full amount of taxes plus a fine, which is computed on the amount evaded $fte$. We follow Yitzhaki (1974) and set a penalty on the evaded tax rather than on the evaded amount as in Allingham and Sandmo (1972). In addition, we simplify the model assuming that the random audits, and the resulting penalty, are contemporaneous with the evasion decision while, in the real world, penalties are assessed later (Andreoni 1992). The cost of tax evasion, which is higher the higher is honesty, is distributed according to the cdf $G_\upsilon$, whose pdf is $g_\upsilon$. If the buyer and the seller do not reach a deal\footnote{Even if there are gains from collaborative tax evasion, bargaining might fail in the incomplete information case studied in the appendix.}, or if the seller does not engage in tax evasion at all, the utility is simply equal to $v^0_s = p(1 - t)$. Comparing $v^0_s$ with $v^e_s$ we notice that the cost of cheating is $d + \upsilon$ while the benefit is the evaded amount minus the expected sanction. To make the analysis interesting, we assume that $1 - \pi (1 + f) > 0$, so that a trade off exist. This assumption implies a limit to the audit probability $\pi$ and to the fine $f$. The utility of a buyer in case of cooperation is

\[
v^c_b = (1 - \pi)[u - (p - d)(1 + \vartheta)] + \pi[u - (p - d)(1 + \vartheta)] - s
\]

\[
= u - (p - d)(1 + \vartheta) - s
\]

where $u$ is the utility from purchasing the good, $\vartheta$ is the tax on currency withdrawals and $s$ is the cost of tax evasion or tax morale, which is distributed according to the cdf $G_S$, whose pdf is $g_s$. Since, in order to evade, the transaction must be paid in cash, the buyer must pay $\vartheta$ on the negotiated effective amount of the transaction $p - d$. If the buyer does not cooperate, he must still choose whether to use cash or an alternative payment instrument such as a credit or debit card. In the former case the utility for the buyer is

\[
v^0_b(cash) = u - p(1 + \vartheta - \tau)
\]

Instead, if he chooses an alternative payment instrument the utility becomes
\[ v_b^0(\text{card}) = u - p(1 - \tau) - c \] (3)

where \( \tau \) is the tax rebate and \( c \) is the cost associated with alternative payment instruments. We do not model a cost associated to the use of cash, even if it is clearly bigger than zero. In fact the cash must be withdrawn from ATM machines (Shoeleather cost), stored and protected from the risk of theft. We normalize all these cost to zero for simplicity. An important remark is that the behavior of those individuals who benefit from the use of cards (negative \( c \)) is identical to the one of those who have \( c = 0 \). We denote by \( g_c \) the pdf and by \( G_c \) the cdf of the distribution of the cost \( c \). We also assume that the distributions of \( c \) and \( s \) are independent. If the buyer chooses not to cooperate with the seller and asks for a receipt, he receives a tax rebate on the full amount of the transaction \( p \). In this case, since he is free to choose among different payment instruments, he will choose the one with the lower cost. More formally, cash is preferred to cards if and only if \( c \geq p\vartheta \). From now on, we denote the utility of a buyer who does not cooperate

\[ v_b^0 = u - p(1 - \tau) - \min \{p\vartheta, c\} \]

The tax rebate and the TCW affect the buyer’s incentive to cooperate rather than the terms of the gamble faced by the tax evader. Moreover, both instruments are independent from the probability that the illicit transaction is discovered. This implies that, while risk aversion is crucial in the analysis of the equilibrium effects of fines and enforcement, it is just marginal here.

We model the negotiation between the seller and the buyer over the price as a Nash bargaining. The solution for the bargaining under perfect information is defined by

\[
d^* = \arg \max_d (v^e_s - v^0_s)^{\beta} (v^e_b - v^0_b)^{1-\beta}
\]

s.t. \( v^e_s \geq v^0_s, v^e_b \geq v^0_b \)

where \( \beta \) is the bargaining power of the seller. The solution for the optimal discount is

\[
d^*(v, s, c) = \beta \frac{p(\tau + \vartheta)}{1 + \vartheta} + s - \min \{p\vartheta, c\} + (1 - \beta) \left[ et(1 - \pi (1 + f)) - v \right]
\] (4)
for all \( v \) such that \( \nu_\theta \geq \nu_\theta^0 \), i.e.,

\[
v \leq et(1 - \pi (1 + f)) - d^*(v, s, c)
\]

and for all the couples \( s \) and \( c \) such that \( \nu_\theta \geq \nu_\theta^0 \), i.e.,

\[
s \leq d^*(v, s, c)(1 + \theta) - p(\tau + \theta) + \min \{p\theta, c\}.
\]

The optimal discount is zero in case conditions (5) and (6) do not hold. If the seller finds an agreement with the buyer his utility \( \nu_\theta = p(1-t) + te[1 - \pi (1 + f)] - d^*(v, s, c) - v \) is increasing in the amount evaded. This is because the positive, direct, effect of an increase in the amount evaded \( t [1 - \pi (1 + f)] \) is larger than the indirect negative effect due to the increase in the discount that the seller must concede to the buyer \( (1 - \beta) t [1 - \pi (1 + f)] \), so that \( e = p \).

By plugging the optimal discount (4) into (6) we find

\[
s \leq (1 + \theta) \left\{ et[1 - \pi (1 + f)] - v - \frac{p(\tau + \theta) - \min \{p\theta, c\}}{1 + \theta} \right\}.
\]

### 3.2 Tax evasion

We use the condition (7) to compute the equilibrium level of tax evasion. First, we consider the buyers with \( c < p\theta \) to obtain a threshold value \( \tilde{s}_1(v, c) \) such that all the buyers of type \( c < p\theta \), with an honesty lower than \( \tilde{s}_1(v, c) \) cooperate. Next, we define the level of seller honesty \( \tilde{\nu}_1 \) that makes no buyer willing to cooperate, such that \( \tilde{s}_1(\tilde{\nu}_1, c) = 0 \). Doing the same for \( c > p\theta \), we obtain a second threshold \( \tilde{s}_2(v) \) which does not depend on \( c \) and such that all the buyers of type \( c > p\theta \), with honesty lower than \( \tilde{s}_2(v) \), cooperate. Finally, we define \( \tilde{\nu}_2 \) such that \( \tilde{s}_2(\tilde{\nu}_2) = 0 \). We get the following expression for total tax evasion \( E \) :

\[
E = \int_0^{p\theta} \left( \int_0^{\tilde{\nu}_1} \left( \int_0^{s_1(v,c)} g_s ds \right) g_v dv \right) g_v dc + [1 - G(p\theta)] \int_0^{\tilde{\nu}_2} \left( \int_0^{s_2(v)} g_s ds \right) g_v dv.
\]

\(^6\)Schweizer (1983) and Cremer and Gahvari (1993, 1999), assume that the revenue concealed from the tax authorities by the seller entails an extra cost thus obtaining an interior solution for the amount evaded. We do not include such a cost since we lack the necessary data to assess the increase in expenditure needed to conceal more revenue.

\(^7\)By plugging the expression (4) into (5) instead of (6) we find exactly the same results.
We are interested in the effect of the government policy $P = \{t, \tau, \vartheta\}$ on the total amount of tax evasion $E$. The simplest way to have zero tax evasion is, obviously, to have no taxation at all. However, a Government needs a minimum amount of resources to function and, as shown by the following derivative, the amount of tax evasion is increasing in the tax $t$. This is in contrast to the basic Allingham and Sandmo model, where an increase in the tax rate would reduce evasion, but perfectly in line with most of the empirical literature, that found that a higher tax rate generally leads to less compliance (Clotfelter, 1983; Crane and Nourzad, 1992; Alm, 2012):

$$\frac{\partial E}{\partial t} = \int_{0}^{\tilde{v}_1} \left[ \int_{0}^{\tilde{s}_1(v, c)} p(1 - \pi(1 + f))(1 + \vartheta)g_v dv \right] g_c dc + [1 - G(p\vartheta)] \left[ \int_{0}^{\tilde{v}_2} g(\tilde{s}_2(v))p(1 - \pi(1 + f))g_v dv \right].$$

We now investigate the effect of the TCW $\vartheta$ and of the tax deductions $\tau$ for buyers who ask for a receipt. Both instruments are meant to reduce the incentives to cooperate. This intuition is correct for the tax deduction, as shown by the following derivative:

$$\frac{\partial E}{\partial \tau} = \int_{0}^{\tilde{v}_1} \left[ \int_{0}^{\tilde{s}_1(v, c)} -\frac{pg(\tilde{s}_1(v, c))}{1 + \vartheta}g_v dv \right] g_c dc + [1 - G(p\vartheta)] \left[ \int_{0}^{\tilde{v}_2} g(\tilde{s}_2(v)) - \frac{pg(\tilde{s}_2(v))}{1 + \vartheta}g_v dv \right].$$

The amount of tax evasion is indeed decreasing in the buyer’s deduction $\tau$. However, the effect of the TCW on the total amount of evasion is more complicated, as shown in the next derivative:

$$\frac{\partial E}{\partial \vartheta} = \int_{0}^{\tilde{v}_1} \left[ \int_{0}^{\tilde{s}_1(v, c)} g(\tilde{s}_1(v, c)) \frac{\partial s_1(v, c)}{\partial \vartheta} g_v dv \right] g_c dc + [1 - G(p\vartheta)] \int_{0}^{\tilde{v}_2} g(\tilde{s}_2(v)) \frac{\partial \tilde{s}_2(v)}{\partial \vartheta} g_v dv,$$

where

$$\frac{\partial \tilde{s}_1(v, c)}{\partial \vartheta} = pt(1 - \pi(1 + f)) - v - p < 0,$$
while
\[ \frac{\partial \tilde{s}_2(v)}{\partial \vartheta} = p\tau[1 - \pi(1 + f)] - v > 0. \]

Indeed, the amount of tax evasion can be both increasing or decreasing in the TCW. The threshold \( \tilde{s}_1(v, c) \) such that all the buyers of type \( c < p\vartheta \), with honesty lower than \( \tilde{s}_1(v, c) \) cooperate is decreasing in \( \vartheta \), while the threshold \( \tilde{s}_2(v) \) for types \( c > p\vartheta \) is increasing. The reason is that, if the buyer does not cooperate, he must still choose whether to use cash or not and cash is better if and only if \( c \geq p\vartheta \).

For high \( c \) (\( c \geq p\vartheta \)), buyers prefer to use cash even if they do not want to cooperate with the seller, in order to avoid the cost of electronic payments. This implies that the TCW does not impose an extra cost for cooperation, making cooperation more attractive: a buyer who cooperates pays \( \vartheta \) over the cash needed for the transaction, that is the price \( p \) minus the discount \( d \), while not cooperating the cash needed over which \( \vartheta \) must be paid is the full amount \( p \).

For low \( c \) (\( c < p\vartheta \)) buyers who do not intend to cooperate with the seller will prefer to bear the cost \( c \) and use electronic means of payment. Then, an increase in the TCW makes cooperation relatively more costly.

Finally, since the derivative of tax evasion with respect to the TCW is positive for low values of \( \vartheta \) and negative for high values, an increase in \( \vartheta \) is more likely to decrease tax evasion the larger is \( \vartheta \). In other words, a tax on cash withdrawals is an effective tool to fight tax evasion only if it is set sufficiently high.\(^8\)

We summarize the previous analysis with the following proposition:

**Proposition 1.** A higher tax rate leads to less compliance. A higher tax deduction increases compliance. A tax on cash withdrawals is an effective tool to fight tax evasion only if set sufficiently high.

Importantly, a decrease in \( c \) has the same effect of an increase in \( \vartheta \). Thus an alternative policy to reduce tax evasion entails decreasing the cost associated to managing payment instruments different from cash. However, while the TCW increases the government revenue, decreasing \( c \) is costly and, therefore, infeasible for financially constrained governments. There is also a further

\(^8\) Notice that for \( \vartheta = 0 \) \( \frac{\partial E}{\partial \vartheta} = \int_{\tilde{s}_2}^{\gamma_2} \left[ \int_{\tilde{s}_2(v)}^{\gamma_2} g(\tilde{s}_2(v)) \frac{\partial \tilde{s}_2(v)}{\partial \vartheta} \right] g_v \, dv > 0 \) while for \( \vartheta = 1 \) \( \frac{\partial E}{\partial \vartheta} = \int_{\tilde{s}_2}^{\gamma_2} \left[ \int_{\tilde{s}_1(c)}^{\gamma_1} g(\tilde{s}_1(v, c)) \frac{\partial \tilde{s}_1(v, c)}{\partial \vartheta} \right] g_v \, dv < 0 \).
difference between the two instruments, as the cost $c$ cannot be compressed for all individuals, even in case of high government expenditures. Indeed, while part of the cost are fees charged by banks and can be compensated with a subsidy; the remaining part is made up of psychological (loss of privacy) and cognitive costs (financial literacy), which are difficult to reduce or, worst, eliminate.

### 3.3 Net Government Revenue

We are also interested in the effect of the government policy $P = \{t, \tau, \vartheta\}$ on the tax proceedings. The expression for the government revenue is

$$\int_0^{\vartheta} \left[ p\pi t(1 + f) + (p - d^*(v, s, c))\vartheta \right] \left[ \int_0^{\tilde{v}_1(v, c)} \left( \int_0^{\tilde{s}_1(v, c)} g_\vartheta dv \right) g_\vartheta dv \right] + p(t - \tau) \left[ 1 - \int_0^{\tilde{v}_1(v, c)} \left( \int_0^{\tilde{s}_1(v, c)} g_\vartheta dv \right) g_\vartheta dv \right] g_\vartheta dc$$

$$+ [1 - G(p\vartheta)] \left\{ [p\pi t(1 + f) + (p - d^*(v, s))\vartheta] \left[ \int_0^{\tilde{v}_2(v)} \left( \int_0^{\tilde{s}_2(v)} g_\vartheta dv \right) g_\vartheta dv \right] + p(t - \tau + \vartheta) \left[ 1 - \int_0^{\tilde{v}_2(v)} \left( \int_0^{\tilde{s}_2(v)} g_\vartheta dv \right) g_\vartheta dv \right] \right\}$$

The first line represents buyers with low cost of managing electronic means of payment ($c < p\vartheta$). Those buyers are either matched with cooperating sellers (first term) or not (second term). In case of cooperation and audit, the seller is forced to pay the full amount of taxes plus a fine, which is computed on the evaded amount $tp\pi(1 + f)$. Moreover, since there must be a cash payment, the buyer pays also the TCW, which amount to the rate $\vartheta$ times the negotiated amount of the transaction $p - d$. When the matching does not lead to tax evasion, the government revenue is the tax paid by the seller net of the tax rebate for the buyer, $p(t - \tau)$.

The second line represents buyers with high cost of managing electronic means of payment ($c > p\vartheta$). In case of cooperation (first term), the government cashes in exactly the same amount $p\pi t(1 + f) + (p - d^*)\vartheta$ as previously described. Conversely, when the matching does not lead to tax evasion (second term), the revenue is $p(t - \tau + \vartheta)$, since the government collects the TCW also on those honest buyers who prefer to use cash because of their high cost of using alternative means of payment. The TCW levied on those individuals represents a pure transfer to the State and should be reimbursed if we want to leave their purchasing power unchanged.

Moreover, introducing a tax on cash withdrawals imposes a cost on those honest buyers (those with $c < p\vartheta$) who opt for alternative means of payment. This cost, differently from the TCW $\vartheta$ is
not a transfer from the citizens to the State but a loss for Society as a whole, and is equal to
\[
\int_0^{\rho} c \left[ 1 - \int_{v_1}^{\tilde{v}_1} \left( \int_0^{\tilde{s}_1(v,c)} g_s ds \right) g_v dv \right] g_c dc.
\]

Since \( c \) is measured in monetary equivalents, it is possible to subtract it from the government revenue\(^9\).

Finally, while for simplicity we ignore the costs associated with different payment instruments for buyers, sellers and for the society as a whole, we recognize that the shift toward a cashless society, induced by the TCW, saves resources and appears to be beneficial also in terms of cost savings (see Humprey et al., 2003 and Garcia-Swartz et al., 2006). It is therefore likely that our analysis underestimate the benefit from a TCW.

4 Numerical Analysis

We now consider empirically plausible distributions for honesty or tax morale and for the cost of managing payment instruments different from cash. Since we do not have an analytical solution for those distribution, we resort to a numerical solution of a calibrated version of the model.

In the next section we summarize the parametrization and calibration which define what we call the “Prototype Economy”. We then proceed by illustrating the results for this fictitious economy. We have two reasons that motivate this approach. First it is difficult to identify, with a reasonable amount of confidence, the exact value of some of the relevant empirical measures, like the current level of tax evasion, that we need to calibrate the model precisely. This forces us to consider a wide range of parameters. Second, we do not want to focus exclusively on a specific country, but rather to provide the widest possible perspective on these issues.

In Section 4.2 we highlight the comparative statics, including the robustness to different, alternative, parameterizations and calibrations.

In Section 4.3 we identify the optimal policy. We consider the net government revenue as a policy objective, preferring it to the gross government revenue and to a measure of welfare (Cowell 1990, Sandmo 2005). The reason why we subtract the cost of credit cards from the total revenue is

\(^9\)Notice that our results are not affected by a mass of buyers with negative cost from using electronic means of payment since they would have chosen those alternative means of payment irrespectively from the policy.
because, as already stressed, the Government, by imposing a tax on cash $\vartheta$, forces some individuals, who do not cooperate with tax evaders, to bear a cost. We exclude the welfare measure for two reasons: first, it would entail choosing whether to include or not the utility of tax evaders and cooperating buyers; second, in our model, taxation is a waste of resources since there is no public good financed with tax receipts and enjoyed by the agents. In such a context, including tax evaders might lead to strange policy outcomes: indeed facilitating evasion will improve the situation of evaders and force non evaders to evade. Including a public good in the model, on the other hand, would obscure the functioning of the basic mechanisms. Moreover, we lack the necessary data to assess how tax revenue is transformed in the public good.

In Appendix A we also propose an example of the optimal policy for a real world country, Italy, a country where tax evasion is widespread and unanimously considered as one of the biggest national problems.

In all the exercises we assume that the government can freely use all the instruments that we consider and that there are no administrative costs or side effects that must be accounted for. In other words, we abstract from problems associated with the implementation of the policies. While this might be innocuous for the tax rate and the tax rebates, which are typical in almost all countries, it is certainly not so for the tax on cash, which is far more challenging to implement. Since, as it will be shown later, the higher the rate of the tax on cash the higher its implementation problems, we also discuss the optimal policy in case the maximum feasible cash tax rate is 5%.

4.1 The Prototype Economy

We start the numerical analysis by choosing a set of parameters and calibration targets that define the baseline, fictitious, prototype economy.

For the tax rate we consider two different choices associated with two different model interpretations. On the one hand, we can interpret the tax rate as the (marginal) tax rate on the firms revenues or profits (which in our simple model coincide). Another possibility is interpreting the tax as a sale tax. Yet another possibility is an interpretation as a value added tax, in which case the price $p$ must be interpreted as the value added of the transaction instead of the transaction price. Our benchmark choices are, respectively, $t = 0.3$ for the income/profit interpretation and...
\( t = 0.1 \) for the sale tax/vat interpretation.

For enforcement probability and fine we choose \( \pi = 0.01 \) and \( f = 0.5 \) respectively. In the model, we admittedly take a shortcut by assuming a random and constant probability that does not depend on the seller’s characteristics and on the evaded amount. It is certainly true that a big firm that evades 90% of its profits faces a higher audit probability than a small, less visible, business that seldom evades a small 10% (Yitzhaki 1987). We also abstract from congestion effects in law enforcement (Galbiati and Zanella 2012), which imply that, for a fixed amount of government resources devoted to enforcement, the individual audit probability decreases the higher the number of individuals that evade. This is the reason why we perform some robustness test on the probability, which can be also interpreted in terms of the size and characteristics of the firm, with higher probabilities corresponding to bigger, more visible, businesses or past evaders. We do not consider the auditing probability as a policy instrument since we do not really have a cost of enforcement in the model and there is no clear way of introducing one\(^{10}\).

Since there is only one country (Pakistan) with a tax on cash withdrawals, we set \( \vartheta = 0 \). Next, we normalize the cost of using cash to zero. Provided that the only individuals who do not use cash if \( \vartheta = 0 \) are the individuals with \( c = 0 \) (or negative), we need a probability distribution with a mass in 0. Moreover we need to make an assumption about the shape of the distribution: we consider an exponentially declining probability mass for higher cost values representing a society in which most of the individuals have a zero or a very small cost of using payment instruments different from cash, but where a small mass of individuals has a high cost (think about the elderly, for instance). The distribution that we use is the following mixture:

\[
g_c(x, \lambda) = \begin{cases} 
0 & \text{Prob } \lambda \\
\lambda e^{-x\lambda} & \text{Prob } 1 - \lambda.
\end{cases}
\]

In the prototype economy we set \( \lambda = 0.2 \), or 20% of the population that does not use cash for the transactions. Of course we test the robustness of the model results to alternative assumptions about the cost \( c \), in particular to the possibility of a different (higher) value of \( \lambda \).

\(^{10}\)Reinganum and Wilde (1985) highlight the optimal auditing rule of the tax authority. Slemrod, Blumenthal, and Christian (2001) and Kleven et al. (2011) study the effects of the threat of enforcement on reported income using field experiments.
We set $\beta = 0.5$ since we have no particular reason to assign a higher bargaining power to the buyer or to the seller, but we discuss the robustness of the model results. The values of $p$ and $u$ are just scalings, and do not affect the main findings. We set $p = 10$ and $u = 1.5p$. Of course if $u$ is not sufficiently higher than $p$, there is the possibility that an effect of the policy is discouraging the buyer from buying the good, but we rule out this possibility by choosing a high value of $u$. In practice, the number of individuals that uses alternative payment instruments is bigger the bigger is $p$. Ideally we need a model where the cost $c$ is decreasing in $p$, but this is impractical from an empirical perspective, given that we don’t have the detailed information needed to parameterize a whole function. Nevertheless, by considering different levels of $\lambda$, we can implicitly take into account this variability: a higher (lower) $\lambda$ is typical of sectors with higher (lower) average transactions values. Therefore the robustness of the comparative static results to different $\lambda$ must be interpreted as robustness across different sectors of the economy with different transaction values. Notice that, since the price $p$ is just a scaling, we do not change it when we perform this robustness tests (changing it will only deliver different calibrated parameters but exactly the same results).

For the distribution of tax morale, we consider an extremely versatile distribution that assigns values in an interval, the Kumaraswamy distribution, which is essentially a BETA distribution with a different parametrization. The pdf is the following:

$$g(x; a, b, \bar{x}) = \frac{ab}{\bar{x}} \left( \frac{x}{\bar{x}} \right)^{a-1} \left[ 1 - \left( \frac{x}{\bar{x}} \right)^a \right]^{b-1} \quad 0 < x < \bar{x}$$

depending on the value of the parameters, we can have an increasing pdf with most of the probability mass corresponding to high values of tax morale, a decreasing pdf, where the opposite is true, or a peak corresponding to intermediate values. We consider the same distribution of tax morale for both the buyer and the seller $(\bar{s} = \bar{\nu})$, since there is no theoretical or empirical reason to believe that the sellers (for instance) are, a priori, more honest than the buyers.\(^{11}\)

\(^{11}\)Obviously we can argue that the profession choice is also dictated by the different opportunities to evade taxes, so that less honest individuals, more prone to tax evasion because of moral considerations, typically choose to work where it is easier to evade taxes (Pestieau and Possen 1991). Nevertheless, since there is no robust empirical evidence that confirms these speculations (Parker 2003), we decided to abstract from these issues.
(WVS henceforth). This survey is part of an ongoing Worldwide research project whose goal is to compare several aspects of culture among different countries. Among the questions administered to a significant number of individuals in several different countries, there is one specifically related to tax morale, namely “Do you consider justifiable cheating on taxes?” Respondents are asked to pick a number between 0 and 10, where 0 means always justifiable while 10 never justifiable. We consider the average frequencies of the responses to the question, where the average is with respect to all participating countries, and not weighted. We choose to take a simple average because the shape of the empirical distribution of the answers is surprisingly similar across countries, with very few exceptions: a big mass of individuals that never justifies evading and a rapidly declining probability mass.

The core of the calibration procedure is choosing the parameters $a$, $b$, together with upper bound $\bar{s} = \bar{v}$ to match the empirical shape of the distribution of the answers and to match the observed level of tax evasion. We run a simple grid search procedure: for each upper bound of tax morale $\bar{s} = \bar{v}$ we divide the interval between 0 and $\bar{s} = \bar{v}$ into 9 equally spaced subintervals. We consider the threshold values of these intervals as corresponding to the 1-10 scale of the answers of the WWS. We then take couples of $a$ and $b$ and, for each couple, we compute the value of the model-based distribution at the threshold values. We then compute, for each couple, the sum of square distances between the model based distribution and the empirical distribution, which is equal to the observed average relative frequencies from the empirical answers to the questionnaire. We choose $a$, $b$ and $\bar{s} = \bar{v}$ to minimize this sum of square residuals for the target calibrated level of tax evasion, so to have the closest possible match between the model and the data. For a target evasion level of 30%, we end up with $b = 1$, $a = 5.93$ and $\bar{s} = \bar{v} = 1.748$ for $t = 0.3$ and $b = 1$, $a = 5.93$ and $\bar{s} = \bar{v} = 0.551$ for $t = 0.1$

The 30% baseline choice for the evasion level is in line with what Pissarides and Weber (1989) find in the UK for self employed individuals, but it is sensibly smaller than the 57% tax evasion on business income for self employed individuals in the US (Slemrod 2007, using data from the Tax Compliance Measuring Program implemented by the IRS). As robustness checks we also consider two alternative scenarios of, respectively, high tax evasion (50%), and low tax evasion (15%).
4.2 Comparative Statics

To understand the functioning of the model, we start by describing the comparative statics of the three main policy instruments in the baseline model specification. In Section 4.2.1 we describe the main results, which are also plotted in figures (1), (2), (3) and (4). In Section 4.2.2 we explore the robustness of the results to different model parameterizations and calibrations. In all exercises we focus attention on one instrument at the time and, therefore, we set the others to their baseline values.

4.2.1 Comparative Statics: The Prototype Economy

Figure (1) reports the comparative static for different tax rates and for different calibration targets of the tax evasion level, 15%, 30% and 50%. In the left panel we report the tax evasion rate and in the right panel the normalized net government revenue, which is equal to 100 in the benchmark model parametrization for each evasion level. This normalization implies that, subtracting 100 from the net government revenue values, we have their variation with respect to the benchmark. It also implies that the three lines are not expressed in the same unit and, therefore, they are not directly comparable. Evasion is increasing in taxes and we have a standard Laffer curve for the government revenue: an increase in the tax rate determines an increase in tax evasion, with less individuals that pay taxes, but the non evaders that optimally choose to pay taxes pay more. The numerical result tells us that the second effect prevails for low tax rates but, after a threshold, when the number of evaders is sufficiently high, the first effect prevails and the revenue is decreasing in the tax rate. The picture also shows that this threshold, which is also the tax rate that maximizes the revenue, is smaller the higher the starting level of tax evasion\(^\text{12}\). The intuition is that the second effect described above, that is the increase in revenue from non-evaders resulting from the tax increase, is smaller the higher the prevailing tax evasion level. While, as shown in the left panel, the first effect, which is measured by the slope of the tax evasion does not depend on tax evasion level.

Figure (2) reports the effect of the tax rebate \(\tau\). As shown in the left panel, evasion decreases with \(\tau\), since a tax rebate reduces the incentives to cooperate for the buyer. There are two

12\footnote{The higher peak, in the right panel, associated with the higher tax evasion scenario is just a by product of the scaling of the graph.}
contrasting effects on the government revenue: on the one hand, the decreasing evasion increases revenue. On the other, the higher \( \tau \) the higher the transfer from the government to the non-cooperating, honest, buyers. The right panel shows that there is a threshold level for the tax rebate such that, if the rebate is below the threshold, the first effect prevails and the revenue is increasing in the rebate; viceversa, if the rebate is above the threshold, the second prevails and the revenue is decreasing in the rebate. The picture also shows that this threshold value, which is also the one that maximizes the government revenue, is higher the higher the prevailing tax evasion level. For a calibrated tax evasion level of 30\%, the optimal tax rebate is \( \tau = 3.5\% \). If the calibrated tax evasion rate was, for instance, 50\%, the optimal \( \tau \) would be 6.5\%, while it would be only 2\% in case of a 15\% evasion level. The reason is that, since an increase in the rebate determines an increase in the total transfers from the government to the (already) honest taxpayers, the cost of increasing the rebate to fight tax evasion is higher the smaller the tax evasion level. It is not optimal to fight small levels of tax evasion using a tax rebate.

Figure (3) reports the effect of the TCW \( \vartheta \). As previously stressed, there are two effects on tax evasion of an increasing TCW rate: for the individuals with high cost \( c \) (the cash users), increasing \( \vartheta \) increases the incentive to cooperate, increasing tax evasion. For the individuals with low cost \( c \) (the card users), increasing \( \vartheta \) decreases the incentive to cooperate, decreasing tax evasion. In addition, the number of card users increases with the TCW rate and the number of cash users decreases. The picture shows that evasion is first increasing in the TCW and then decreasing. Basically there must be a sufficiently high number of card users for the second effect to prevail on the first and, therefore, keeping the characteristics of the economy fixed, a sufficiently high TCW rate that increases such a number. In addition, the higher the prevailing tax evasion rate, the smaller the TCW rate above which tax evasion is decreasing. This is because if the tax evasion is low, there is a big mass of non collaborative buyers that are cash users. The effect of the TCW on the net government revenue is twofold: on the one hand an increase in the TCW rate affects the cooperation rate and, therefore, the level of tax evasion. On the other, it affects the total cost of credit card use that must be subtracted from the gross revenue. For low levels of the TCW rate, cooperation and, therefore, evasion, is increasing, which translates in a decreasing gross revenue. Since more individuals are using cash because they are evading, there is a lower total cost of credit card use. Thus the net revenue can be increasing even in case of an increase in tax evasion.
Viceversa, for high values of the TCW, tax evasion is decreasing, but the net government revenue can be still decreasing because the total credit card cost is very high. Tax evasion is lower than the benchmark for a very high tax on cash, but the net government revenue might be lower or higher than the benchmark for such values. In addition, the gain in government revenue obtained with the optimal tax rate is higher the higher the starting level of tax evasion. Again in case of a low rate of tax evasion there is a big mass of non collaborative buyers that use cash and the increase in tax evasion resulting from the TCW increase is not matched with a sustained decrease in the total cost of credit card use, with a government revenue that can also be decreasing in the TCW. In fact there is no gain in revenue from the introduction of the TCW if the starting evasion rate is sufficiently low.

Summing up, we have a numerical analog of Proposition 1 but also some additional results. We summarize all the previous results in the next proposition:

**Proposition 2.** i) Evasion is increasing in taxes and we have a standard Laffer curve for the government revenue. The optimal tax is smaller the higher the baseline tax evasion; ii) A moderate tax rebate can increase government revenue and reduce evasion. The optimal tax rebate is higher the higher the baseline tax evasion; iii) The tax on cash withdrawals that maximizes net government revenues is higher the higher the baseline tax evasion.

In Figure (4) we plot the joint effect of tax rebates and TCW for a fixed tax rate level equal to 30%. From the left panel, it is easy to see that, for a fixed tax rebate, there is a hump shaped response of the TCW that maximizes the net government revenue. Similarly, fixing the TCW there is hump shaped response of net revenue to tax rebates. The interaction of these two results implies that there is an interior solution when maximizing the net government revenue. Tax evasion is low for high tax rebates and it responds less to TCW the higher the tax rebate.

### 4.2.2 Comparative Statics: Robustness

We now consider various robustness exercises. We start from a higher target value for the credit card users, \( \lambda = 0.5 \). In this scenario, tax evasion is always decreasing in \( \vartheta \) and, for a 30% benchmark calibrated evasion rate, \( \vartheta = 0.15 \) maximizes the government revenue net of the cost. Since there is a higher mass of individuals with a small cost of using alternative payment instruments, the
government requires a smaller tax on cash to prevent cooperation from the buyers to reduce evasion. Reduced evasion, in turn, raises the tax collection and decreases the collection of the tax on cash withdrawals but the first effect prevails and revenues are increasing.\footnote{Since the use of cash is less common in sectors of the economy where the average transaction amount is higher, variations in $\lambda$ describe the ideal TCW in different sectors (remember that the price $p$ and the value $u$ are just scalings in the model, so we would have exactly the same results if we redid the model parametrization and calibration for a different price level).}

The comparative static results are similar in case of a lower baseline tax rate, $t = 0.1$ for the sale/vat tax interpretation of the model. The only difference is a slightly smaller value of the optimal $\tau$ and $\vartheta$. Changing the bargaining power of the seller $\beta$ results in a different distribution of the gains from evasion but a similar effect of policies on the equilibrium quantities. We also tried a rather extreme value for the enforcement probability, $\pi = 0.3$.\footnote{As already stressed in the previous section, for some businesses the marginal probability of detection can be very high, for instance in case of big firms with high level of evasion or in case there has already been evasion in the past.} In this scenario, the comparative static results are similar, except that the optimal levels of $t$, $\tau$ and $\vartheta$ that maximize the net government revenue (everything else equal) are now smaller. Therefore enforcement is a substitute for all these policies. In our model, enforcement is costless, so we cannot really evaluate the impact of enforcement on the government revenue and, therefore, we cannot single out the optimal enforcement level.

We also tried risk aversion, assuming a CRRA utility function for both the seller and the buyer and risk aversion parameter equal to 3. The comparative static results are essentially similar to the benchmark. As already said, the reason is that the tax rebate and the TCW affect the buyer’s incentive to cooperate rather than the terms of the gamble faced by the tax evader.\footnote{We also consider the comparative static results with respect to the fine $f$. Overall a higher fine results in a smaller level of tax evasion and in a higher government revenue, but the quantitative effect is very small. If $f = 3$, ten times bigger than the baseline value, evasion is only 1\% lower than the benchmark and the revenue 2.5\% higher.} The previous results are summarized in the following proposition:

**Proposition 3.** i) The larger the mass of individuals with high cost of using payment instruments alternative to cash (the smaller $\lambda$), the higher the optimal tax on cash withdrawals; ii) Tax rebate and tax on cash withdrawals are a substitute for enforcement; iii) Risk aversion is crucial in the analysis of the equilibrium effects of fines and enforcement, but marginal in the analysis of the effects of the tax rate, tax rebates and TCW.
4.3 Optimal Policy

We perform two different exercises. The first entails fixing the tax rate to find the optimal combination of $\tau$ and $\vartheta$ to reach a desired objective. The second is finding the optimal combination of $t$, $\tau$ and $\vartheta$ to reach the same objectives. The motivation behind the first exercise is that, since we are considering an extremely stylized microeconomic model, we are not able to capture all the possible macro effects of a change in the tax rate and, therefore, we are not in a good position to evaluate the consequences of different tax rates.\footnote{For instance, increasing taxes might induce a recession, which is also part of the reason why politicians are typically reluctant to implement them.}

We consider two different possible objectives of the policy. The first is the maximization of the net government revenue. The second entails the maximization of the same objective but conditional on the reduction of tax evasion below 1%. The reason for this second objective is that, as noted by Slemrod and Yitzhaki (1987), the social benefit of tax evasion reductions is not well measured by tax revenue increases only. Additional benefits include, among others, reduced risk bearing, increased efficiency and better competition among businesses. For both policy exercises, we stress the gain in gross government revenue with respect to the corresponding benchmark with fixed tax rate, no tax rebates and no tax on cash. This measure gives an idea of the possibility of the government to compensate honest taxpayers for the side effects introduced by the TCW. We propose some discussion about possible compensation schemes in Section 5.

A caveat before proceeding to the exercises: here we pick some numbers, but we do not intend to take the numbers very seriously. Instead, we think of them as suggestive of the direction that the policy should follow.

In Section 4.3.1 we summarize the optimal policy for the benchmark prototype economy, while in Section 4.3.2 we explore the robustness of the results.

4.3.1 Optimal Policy: The Prototype Economy

Fixing the tax rate at the benchmark 30%, the policy that maximizes the net government revenue is $\tau = 3\%$ and $\vartheta = 15\%$. The level of tax evasion associated to this policy is rather high, 12.5%, even if significantly lower than the 30% benchmark. The gross government revenue is 50% larger than the benchmark. Conditional on keeping evasion below 1%, the policy that maximizes the
net government revenue entails a higher tax rebate and a higher cash tax, specifically $\tau = 10\%$ and $\vartheta = 18\%$. The gross government revenue in this case is just 6\% smaller than in the previous exercise and, more interestingly, 38\% bigger than the benchmark value with no tax rebates and no tax on cash. This result stress that the cost of fighting evasion in term of forgone tax revenue is quite small.

Using also the tax rate as policy instrument, the policy that maximizes the net government revenue entails fixing $t$ and $\vartheta$ as high as possible and $\tau$ to zero. The cost of this policy is a high level of tax evasion, mainly induced by the increase in taxes. Conditional on keeping evasion below one percent, the optimal policy is $t = 24\%$, $\tau = 3\%$ and $\vartheta = 18\%$. Again the optimal policy entails a mix of TCW and of the tax rebate. For this policy the gross revenue is still almost 42\% higher than the benchmark value where no cash tax and tax rebates are used, although significantly smaller than what could be achieved without fighting evasion. Nevertheless we still find that tax evasion can be eliminated while generating an hefty amount of extra tax revenue. The following proposition summarizes the main result of the optimal policy exercise:

**Proposition 4.** An appropriate mix of tax rebates and cash tax can curb tax evasion while, at the same time, raising additional tax revenue.

This additional revenue can be also rebated back to the taxpayers, to compensate the side effects induced by the TCW.

The TCW is arguably difficult to implement and more so the higher the rate (see Section 5 for a discussion). Therefore we perform an additional exercise: we find the optimal policy conditioning on keeping the tax on cash below 5\%. The optimal policy to maximize tax revenue is $t = 30\%$, $\tau = 3\%$ and $\vartheta = 5\%$, with a 9\% evasion and a 33\% bigger gross government revenue. Conditional on keeping evasion below 1\% the optimal policy entails the same tax on cash and tax rebate but a smaller tax rate, $t = 27\%$, for a 26\% bigger revenue. Thus it is still possible to curb tax evasion and raise additional resources with the only difference of a more modest increase in government revenues. Proposition 4 still holds.
4.3.2 Optimal Policy: Robustness

We now consider four robustness exercises: a higher level of tax evasion, 50%; economies with less widespread use of cash, \( \lambda = 0.5 \); a lower tax rate, \( t = 0.1 \); risk aversion for both the seller and the buyer, with a CRRA utility function and a risk aversion parameter of 3. The first exercise is meant to be representative of countries where tax evasion is higher. The second exercise is representative of countries with more developed and competitive financial systems or with a more financially literate population, where the use of electronic currency is more widespread; alternatively, it is representative of sectors of the economy where, given the higher value of the typical transactions, using cash is less practical. The third exercise is designed for the sale/vat tax interpretation of our tax evasion model.\(^{17}\)

We start from the higher baseline tax evasion, 50%. Again the conclusion is that a mixture of cash taxes and tax rebates can virtually eliminate tax evasion and raise a substantial amount of additional resources. The additional piece of information that we have with this robustness check is that, if the tax evasion is already very high, the tax rebate must be higher.

The second robustness check entails a higher use of payment instruments alternative to cash, with \( \lambda = 0.5 \). The conclusion from this exercise is that the tax on cash must be smaller the smaller the cost of using payment instruments alternative to cash. Therefore in sectors of the economy where the typical transaction amount is bigger, so where it is easier to avoid paying cash, the optimal policy would entails a lower TCW.

The third robustness check entails the sale tax interpretation of the model, with \( t = 0.1 \). Again it is possible to curb evasion while raising a substantial amount of additional government resources. The difference with the benchmark model parametrization is a lower level of the tax rebate, which is a consequence of the lower tax rate from which we started.

The final robustness test entails risk aversion for both the seller and the buyer. Instead of the linear utility of the baseline model, we use a CRRA utility function with risk aversion parameter equal to 3. We found that the optimal policy is almost indistinguishable from the prototype economy, with only very small differences in the value of the optimal policy instruments. Once again, our main policy instruments, the tax on cash and the tax rebate affect significantly the

\(^{17}\)The exact figures of the following exercises are available upon request.
buyer’s incentive to cooperate rather than the terms of the gamble between that the tax evader faces. As in the prototype economy, it is still possible to raise additional government revenue while reducing evasion below 1% even with a 5% upper bound to the TCW. If it is not possible to use the TCW, however, the government revenue associated to the zero tolerance policy is almost equal to the non policy benchmark.

5 Cash Tax Implementation

In this section we discuss some challenges to the implementation of the tax on cash withdrawals and propose solutions to partially overcome them. Since it is a tax on cash withdrawals from a bank account, either a checking or a saving account, it can be naturally implemented by banks. The bank can collect the tax from the public and then transfer the money to the tax administration, with or without a documentation that might include the identity of the individual and the amount of the transaction.

The first, and foremost, challenge to the implementation of the cash tax is the possible emergence of a parallel cash economy: both sellers and buyers might start using whatever cash they already have for the transactions and accumulate it to avoid using the banking system and paying the TCW.

To credibly implement the TCW, the government must stifle the cash economy.

The first simpler possibility is to keep the cash tax rate small, in order to drastically reduce the risk of a bank run and to downsize the dimension of the parallel cash economy. We already stressed that, even with this constraint, it is still possible to reduce tax evasion and raise additional tax revenue, although less than in the unconstrained case.

A second possibility is the contemporaneous introduction of a tax on cash bank deposits, together with a ban on cash purchases of financial instruments (treasury bonds, corporate bonds or stocks) and of sanctions for the individuals caught exchanging cash amounts above a certain threshold. The idea is to lower the incentives of the sellers to accept cash payments and hoard cash, making difficult to use it. Consider, for instance, a small shopkeeper that evades taxes by selling for cash. He can use the accumulated cash to pay the suppliers and to pay the wages to the employees, fostering the cash economy. But he can also save them, perhaps for the down payment
of a car or house or perhaps to improve the shop in the future. The sanctions for cash transactions of big amounts are meant exactly to limit the use of this accumulated savings. In addition, a ban on cash purchases of financial instruments greatly reduces the possibility to accumulate savings. A rational investor will decide to deposit if the tax is lower than the capitalized interests from the financial instruments. Overall, these sellers see a reduction in the value of their savings for all the cash transactions and this greatly reduces their incentive to accept cash payments (Morse et al. 2009).

A second, related, challenge concerns the dynamic of the introduction of the tax. If the tax is announced and then implemented, it is likely that a bank run will take place, with individuals withdrawing cash to avoid paying the tax and/or to facilitate tax evasion in the future.

The probability associated with these two scenarios is higher the higher the TCW. It is difficult to think of a massive bank run as a result of a small TCW, since the cost of managing all the payments in cash will most likely be higher than the cost imposed by the tax. In addition, operating exclusively in the cash economy hampers or precludes the possibility of obtaining mortgages or even short term financing for the sellers, imposing a very high cost to them (Straub 2005, Antunes and Cavalcanti 2007, Gordon and Li 2009, Capasso and Jappelli 2013). Also, some credit card issuers already charge a small fee for ATM withdrawals, but many individuals use them anyway.

A third problem is related to the costs of the electronic payment system. One the one hand, the TCW induces many individuals to pay the cost of using debit and credit cards. On the other, the sellers might accept electronic payments to avoid loosing customers (in favor of establishments where they can pay with credit cards) and, since this services are costly, they will face lower profits. In the end, the TCW might create rents for the banks. To neutralize this effect part of the extra government revenue can be used to provide tax rebates to consumers and sellers that help pay for the credit and debit card fees.

More generally, compensating honest taxpayers (both sellers and buyers) that do not use cash is a crucial element of our policy proposal, since it will boost its acceptability by the general public, maximizing the probabilities that it will be successful. If it is perceived as unfair, on the other

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18Garcia Swartz et al. (2006) shows that cash remains the cheapest method for grocery store and for electronics store to accept.

19To avoid overpricing of the bank’s financial services, the Government can also regulate the market imposing maximum prices or other regulatory schemes.
hand, it is more likely doomed to be ineffective, as it will strengthen a social norm against it (Bordignon 1993, Falkinger 1995, Torgler 2003 [1], Slemrod 2003). Particularly challenging is to design the compensation for those who have a cost of using the electronic payment instruments other than the banking fees. We think in particular about specific categories of individuals, like the elderly, who, being less familiar with new technologies, might find it hard to cope with an electronic payment system. However, it is difficult, both from a technical and from a legal perspective, to link a monetary compensation to the cost, since beside old age and disability, there is not really much more information that can help discriminate between individuals.

A fourth implementation problem is specific to currency areas: the TCW should be implemented in all the countries at once and, to avoid arbitrage, the tax rate should be the same in all countries. This will hamper the possibility of tailoring the policy to the specific needs of a country. One plausible argument against this objection is that the possibilities of arbitrage are not widespread. For individuals that typically live and work in one country only, that is the majority of them, there is not really the possibility of frequent cross border travels just to withdraw cash or, alternatively, the possibility of opening a bank account in another country. Probably the cost of these operations, which includes travel expenses and banking fees, will reduce the gain from arbitrage at least for a small tax.

6 Incomplete Information

One of the simplifying assumption of our model is the complete information on both sides of the market: the buyer perfectly knows the tax morale of the seller and the seller knows perfectly both the tax morale of the buyer and his cost of using payment instruments alternative to cash. This assumption is reasonable for repeated interactions. Think about the relationship with a family doctor or with the neighborhood grocer, for instance. However it is hardly representative of all market transactions, that often are occasional. To analyze how the results change in a more reasonable incomplete information scenario, we consider a different model set-up following the double auction model of Chatterjee and Samuelson.

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20 The compensation is also crucial since many of the optimal policies that we isolated feature a TCW higher than the tax rebate and this might decrease the purchasing power of the individuals if they perform many expenses in cash.
We consider a seller and a buyer that submit a bid for a price discount, respectively $\psi_s(v)$ and $\psi_b(s, c)$. If the bid submitted by the seller, which is the maximum he is willing to accept to induce cooperation from the buyer, is bigger than the bid submitted by the buyer, which is the minimum he is willing to accept to allow tax evasion, then we assume that there is cooperation and tax evasion, with a discount that is a weighted average of these two bids. Thus if $\psi_s(v) \geq \psi_b(s, c)$ then 
$$d = \beta \psi_s(v) + (1 - \beta) \psi_b(s, c).$$
In this case the seller fully evades taxes and the buyer cooperates, accepting the discount. If $\psi_s(v) < \psi_b(s, c)$ there is no deal, no tax evasion and the utilities of both seller and buyer are unaffected. Instead, in case of evasion, the seller’s expected profit becomes

$$V_s^e = p(1 - t) + \{pt(1 - \pi) - [1 - \pi(1 - ft)]E[d|\psi_s(v) \geq \psi_b(s, c)] - v\} \text{Prob}[\psi_s(v) \geq \psi_b(s, c)]$$

(10)

while the buyer’s is

$$V_b^e = u - p(1 - t) - \min[p\vartheta, c] + \{(1 + \vartheta)[d|\psi_s(v) \geq \psi_b(s, c)] - p(\tau + \vartheta) + \min[p\vartheta, c] - s\} \text{Prob}[\psi_s(v) \geq \psi_b(s, c)].$$

(11)

Both the seller and the buyer choose the bid that maximizes the expected revenue. A pair of best response offer strategies constitute a Nash (or Bayesian) equilibrium. By definition, neither player can increase his expected profit by unilaterally altering his chosen strategy.

In case of uniform distribution of $v$, $s$ and $c$ there is a simple analytical expression for the equilibrium strategies and discount. In Appendix B we discuss such result. However, for the actual distributions that we use, we don’t know the analytical expression for the optimal strategies. Therefore we consider polynomial approximations. The solution algorithm entails guessing an optimal strategy for the sellers, which is a polynomial in his type $v$, to then compute the best response for the buyers, which is a function of both $c$ and $u$. Given this best response, we compute the buyer’s best response and then the difference between this last function and the guessed strategy (we consider the sum of square distances as a metric). We repeat this procedure for different guesses, in the form of different parameters of the polynomial, and we stop when we find the Bayes-Nash equilibrium.

Overall the numerical results are substantially similar to the complete information case, with only two major differences. The first is that the gains from trade are distributed differently.
between the buyer and the seller. Second, the uncertainty about the identity of the buyer in terms of the cost $c$ leads the seller to submit a bid based on the expected, not actual, cost $c$, so the individuals with high cost in a sense matter less for the model equilibrium. Therefore under asymmetric information we have results that resemble the one that we would have obtained under complete information but with a higher value of $\lambda$ or, in general, with a distribution with lower probability mass on high cost $c$ individuals. For instance, we mentioned in the previous section that if the measure of individuals with high cost $c$ is small, evasion is always decreasing in the cash tax. Under incomplete information, evasion is decreasing in the tax cash also with a mixture exponential distribution of $c$ and a small $\lambda$, a scenario that delivered an inverse u-shape relationship under complete information.

7 Conclusion

We presented a model of collaborative tax evasion where a buyer negotiates a price discount with a seller in exchange for not asking the receipt and paying in cash, allowing tax evasion. We study how a tax rebate for the buyer and a tax on cash withdrawals (TCW) affect tax evasion and government revenue. A small tax deduction can be effective at both reducing tax evasion and increasing government revenue and the deduction rate must be higher the higher the evasion rate in the country and the higher the tax rate. The TCW is effective at reducing evasion and increasing revenue only for high rates, and the rate must be higher the higher the evasion rate in the country and the larger the mass of individuals using cash as a paying instrument. We found that an appropriate mix of tax rebates and taxes on cash withdrawals, together with an appropriate choice of a tax rate, can curb tax evasion while, at the same time, raising additional revenue. The additional revenue generated by this policy can be partly used to subsidize the use of credit cards and POS machines, to avoid imposing individual costs to buyers and sellers and, therefore, increasing the public support to the policy. To avoid the emergence of a parallel cash economy, we propose to keep the TCW rate at a low level, which will reduce the revenue with respect to the optimal unconstrained policy, but still drastically reduce the tax evasion rate.

Another way to summarize our results is saying that an abrupt and forced move towards a cashless economy, realized with a tax on cash withdrawals, is a good way to fight tax evasion. In
the following we discuss some alternatives to our anti-tax evasion scheme.

A first obvious alternative would be enforcing a ban on cash transactions, perhaps establishing a threshold value below which it is allowed to use cash. However, the main drawback of this first alternative seems the increased enforcement cost. Importantly, the TCW is intrinsically different from a ban on non traceable payments instruments that, although ideal from the tax authority perspective, would most likely be upsetting to many: first because it would entail a loss of privacy; second because imposing the use of credit cards, cheques or bank transfers for transactions of small amount, can be too cumbersome. In this perspective, the TCW can be seen as putting a price on privacy and transaction ease: it allows to use non traceable payment instruments, to ensure privacy or to speed up the transaction, but there is a cost associated with these benefits.

A less obvious possibility consists in explicit cost-based pricing of payment instruments. Van Hove (2004) argues that under current pricing schemes the use of cash is not discouraged since the fees charged to consumers for making cash withdrawals do not cover the full cost which is recovered through cross-subsidisation.. “In this way, infrequent cash-users de facto subsidise those who make heavy use of cash (including those active in the underground economy) (Van Hove, 2004, p.80”. We see the remedy proposed by Van Hove, that is substituting fees per account with fees per transaction, as complementary to a tax on cash withdrawals.

8 Appendix A: The Case of Italy

In this section we consider actual empirical evidence for Italy as an example of the optimal policy discussed in the main body of the paper. We choose Italy because, beside being the country in which we live and work, it is also one of the countries where tax evasion is particularly pernicious and where it attracts a lot of attention from citizens and media.

For the tax rate we choose \( t = 0.35 \) and the income/profit interpretation of the model\(^{21}\). This number is a (rounded) weighted average of the different rates with weights equal to the percentage of income in the bracket. Obviously we are simplifying, perhaps too much, the complexity of the Italian tax system by picking this number. Firms are in fact required to pay a conspicuous

\(^{21}\)For the sale/value added tax interpretation \( t = 0.21 \), which is the VAT tax rate on most manufacturing goods and personal and professional services. We omit the results for this alternative case as they are a straightforward consequence of what we already discussed in the previous section.
number of taxes: some are fixed and required to stay open, some of which depend on the revenues, some depend on the profits, some on the number of workers. Different assumptions will lead us to include or exclude some of them in the computation of the marginal tax and, inevitably, to different possible numbers and long discussions to defend our assumptions. Instead of doing some complicated computation of the average tax rate, we decided to just stick to this simple benchmark, taking into account that the results that we would obtain in case of a smaller or bigger tax rate can be inferred from the discussion in Section 4.3.

For the enforcement probability $\pi$, we divide the number of tax audits made in 2011 by the “Guardia di Finanza”, the main tax enforcement authority in Italy, by the number of economic units (firms, entrepreneurs, individual professionals) operating in Italy in 2011. There are two kinds of audits implemented by this tax enforcement authority: a more throughout one, which is less frequent but that detects evasion with certainty (a careful screening of all the fiscal documents, together with a detailed analysis of the economic activity) and a more superficial one, much more frequent but less effective (a simple spot control where the agents monitor the day to day activity and step in if there is a violation). In the first case we have $\pi = 0.0067$ while in the second $\pi = 0.172$. We present the results for the first value and we assume that all audits are random and independent on sellers subjective characteristics. While this is certainly true for the spot controls, it is not for the more throughout controls, which are typically the final step of some monitoring activity that takes into account the business characteristics, also based on the past tax reports. For the fine we use the value $f = 0.3$ according to the Italian tax law that prescribes a fine from 6% to 30% to be paid on the evaded amount. In Italy, tax evasion is also subject to jail sentences, in addition to the fines, but only in extreme cases (very high amount), which makes them extremely unlikely. Thus we focus on pecuniary fines only.

Since there is no tax on cash withdrawals in Italy, we set $\theta = 0$. For the distribution of the cost of payment instruments other than cash $c$, we consider data on payment instruments from the ECB (gathered through the national central banks). We divide the sum of all transactions made with credit cards and debit cards by the consumption component of GDP (goods, including durable, and services). We obtain $\lambda = 0.127$. We stick to the assumption of $\beta = 0.5$ and to $p = 10$ and $u = 1.5p$.

To calibrate the model, we need a target value for tax evasion and we need a shape of the
distribution of the tax morale to match. For the tax morale we take the values from the WWS website. For tax evasion, we consider two different sources. The first is the Study by EURES (2012), an Italian independent research institute. Total tax evasion in Italy is estimated to be between 16.3% and 17.5% of the GDP. Both numbers are obtained averaging over different sectors (32.8% agriculture, 12.4% Industry and between 20% and 27% for services) and across different geographic areas. The other source is the ISTAT, the Italian statistical institute, that reports an average of 12.7%, also obtained averaging over different sectors (22% agriculture, 6% industry, 11% construction, 14% services) and geographic areas (9% North, 11% center, 20% South). We set our benchmark close to the average of the two numbers at 15%. We understand that this average can generate a lot of disagreement and, quite frankly, we don’t really know if this is the best possible number that we can use. The main issue here is the interpretation of the model and the sector of the economy for which it is more representative. Perhaps the cooperative tax evasion phenomenon is more widespread in the service sector or among professionals, so we should probably use a higher target level. In addition, there is some direct survey evidence reported by the EURES (2012) that highlights much higher evasion rates among specific professions like, for instance, a 63% evasion rate for baby sitters and housekeepers, 33% for hairdressers, 67% for gardeners etc. Here we present the results only for the baseline average evasion level, but we stress that, in case of a higher evasion level, the same consideration that we made in the robustness test of Section 4.3 apply.

For the 15% benchmark tax evasion level, we end up with $b = 1$ and $a = 5.87$ and $\bar{s} = \bar{v} = 2.025$.

Fixing the tax rate to the benchmark 35%, the policy that maximizes the net government revenue is $\tau = 4\%$ and $\vartheta = 30\%$, with a tax evasion level of 10.7%. and a 45% bigger gross government revenue. Conditional on keeping evasion below 1%, the policy that maximizes the net government revenue entails a higher tax rebate and a lower cash tax, specifically $\tau = 15\%$ and $\vartheta = 15\%$. The gross government revenue in this case is approximately equal to the benchmark value. Again the result is that it is possible to achieve a virtual elimination of tax evasion by using an appropriate mix of cash taxes and tax rebates. Using also the tax rate as policy instrument, the policy that maximizes the net government revenue entails again a high tax rate and a high cash tax and zero rebates, but it induces a high level of tax evasion. Conditional on keeping evasion below 1%, the optimal policy is $t = 25\%$, $\tau = 1\%$ and $\vartheta = 35\%$, with a 37% bigger gross government revenue.
revenue that the benchmark. If the maximum feasible cash tax rate is 5%, the optimal policy is $t = 35\%, \tau = 2\%$ and $\vartheta = 5\%$, with a 13% bigger gross government revenue and a 9% evasion rate. Conditional on keeping evasion below 1%, the optimal policy is $t = 30\%, \tau = 3\%$ and $\vartheta = 5\%$, with both a lower tax rate and a higher rebate. The gross government revenue is equal to the benchmark value. Without using the tax on cash the policy that maximizes the net government revenue entails again higher taxes and higher rebates, $t = 45\%$ and $\tau = 12\%$. For this policy, the gross government revenue is just 1.5% higher than its benchmark value and evasion is slightly more than half the benchmark value, 8%. Conditional on keeping evasion below 1%, the optimal policy is $t = 35\%$ and $\tau = 13\%$, thus a lower tax and a higher deduction. The effect is a 26% lower gross government revenue. In this example, fighting evasion without using the tax on cash is costly.

9 Appendix B: Analytical solution with Uniform Distributions

In this Appendix we propose an analytical solution of the model for the case of incomplete information when the uncertainty is uniformly distributed. Namely $v \sim U[0, \bar{v}]$, $s \sim U[0, \bar{s}]$, and $c \sim U[0, u]$. We show that the solution has a structure similar to the one in the complete information benchmark.

In this section the double auction model of Chatterjee and Samuelson (1983) is used to model the bargaining between the seller and the buyer. Since all the heterogeneity is uniformly distributed, we guess a linear solution for the optimal strategies for both the seller and the buyer and we solve the model by guess and verify. We consider the following strategies

$$
\psi_s(v) = a_1 + a_2 \phi_s(v) \\
\phi_s(v) = \frac{pt(1 - \pi) - v}{1 - \pi(1 - ft)}
$$

(12)

$$
\psi_b(s, c) = a_3 + a_4 \phi_b(s, c) \\
\phi_b(s, c) = \frac{p(\tau + \vartheta) - \min[p\vartheta, c] + s}{1 + \vartheta}
$$

(13)

The seller’s optimal strategy solves the following maximization problem
\[ \max_{\psi_s} p(1-t) + [1 - \pi(1 - ft)] \{\phi_s(v) - \beta \psi_s - (1 - \beta)E[\psi_b(s, c) | \psi_s \geq \psi_b] \} \text{Prob}[\psi_s \geq \psi_b] \quad (14) \]

We have

\[ \text{Prob}[\psi_s \geq \psi_b] = \frac{p_{\theta}}{c} \text{Prob}[\psi_s \geq \psi_b | p_{\theta} > c] + \left( 1 - \frac{p_{\theta}}{c} \right) \text{Prob}[\psi_s \geq \psi_b | p_{\theta} < c] \quad (15) \]

if \( p_{\theta} > c \) the distribution of the buyer’s optimal response is uniform:

\[ \psi_b(s, c) \sim U \left[ a_3 + a_4 \frac{p(\tau + \vartheta) - \tilde{c}}{1 + \vartheta}; a_3 + a_4 \frac{p(\tau + \vartheta) - \tilde{c} + \tilde{s}}{1 + \vartheta} \right] \quad (16) \]

where \( \tilde{c} = E[c | p_{\theta} > c] = p_{\theta}/2 \).

if \( p_{\theta} < c \) the distribution of the buyer’s optimal response is again uniform:

\[ \psi_b(s, c) \sim U \left[ a_3 + a_4 \frac{p\tau}{1 + \vartheta}; a_3 + a_4 \frac{p\tau + \tilde{s}}{1 + \vartheta} \right] \quad (17) \]

Thus

\[ \text{Prob}[\psi_s \geq \psi_b] = \frac{1}{a_4 s} \left[ (\psi_s - a_3)(1 + \vartheta) - a_4 p\tau - a_4 \frac{p_{\theta}^2}{2\tilde{c}} \right] \quad (18) \]

and

\[ E[\psi_b(s, c) | \psi_s \geq \psi_b] = \frac{1}{2} \left[ \psi_s + a_3 + a_4 \frac{p\tau}{1 + \vartheta} + a_4 \frac{(p_{\theta}^2)}{2\tilde{c}(1 + \vartheta)} \right]. \quad (19) \]

Plugging these last two expressions in the seller’s optimization problem and differentiating with respect to \( \psi_s \) we obtain the following first order condition

\[ \psi_s(v) = \frac{1}{1 + \beta} \phi_s(v) + \frac{\beta}{1 + \beta} \left[ a_3 + a_4 \frac{p\tau}{1 + \vartheta} + a_4 \frac{(p_{\theta}^2)}{2\tilde{c}(1 + \vartheta)} \right]. \quad (20) \]

The buyer optimal strategy is the solution to the following optimization problem
\[ \max_{\psi_b} u - p(1 - \tau) - \min[p \vartheta, c] + (1 + \vartheta) \{ \beta E[\psi_s | \psi_s \geq \psi_b] + (1 - \beta) \psi_b - \phi_b(s, c) \} \text{Prob}[\psi_s \geq \psi_b]. \quad (21) \]

The optimal strategy of the seller is uniformly distributed

\[
\psi_s(\nu) \sim U \left[ a_1 + a_2 \frac{pt(1 - \pi) - \bar{\nu}}{1 - \pi(1 - ft)} ; a_1 + a_2 \frac{pt(1 - \pi)}{1 - \pi(1 - ft)} \right] \quad (22)
\]

thus:

\[
\text{Prob}[\psi_s \geq \psi_b] = \frac{1}{a_2 \bar{\nu}} \left\{ (a_1 - \psi_b)[1 - \pi(1 - ft)] + a_2 pt(1 - \pi) \right\} \quad (23)
\]

and

\[
E[\psi_s | \psi_s \geq \psi_b] = \frac{1}{2} \left[ \psi_b + a_1 + a_2 \frac{pt(1 - \pi)}{1 - \pi(1 - ft)} \right]. \quad (24)
\]

Plugging these expression in the buyer’s expected profit and differentiating with respect to \( \psi_b \) we obtain the following first order condition

\[
\psi_b = \frac{1}{2 - \beta} \phi_b(s, c) + \frac{1 - \beta}{2 - \beta} \left[ a_1 + a_2 \frac{pt(1 - \pi)}{1 - \pi(1 - ft)} \right]. \quad (25)
\]

Comparing the first order conditions (20) and (25) with the guesses (12) and (13) we obtain a linear system of four equations in four unknowns \( a_1, a_2, a_3 \) and \( a_4 \). Solving the system and plugging the solution in (12) and (13) we obtain the following optimal strategies:

\[
\psi_s^* = \frac{\beta}{2} \frac{1}{1 + \vartheta} \left( p\tau + \frac{(p \vartheta)^2}{2c} \right) + \frac{1}{1 + \beta} \frac{1}{1 - \pi(1 - ft)} \left\{ \left[ 1 + \frac{\beta(1 - \beta)}{2} \right] pt(1 - \pi) - \nu \right\} \quad (26)
\]

and

\[
\psi_b^* = \frac{1}{2 - \beta} \frac{1}{1 + \vartheta} \left\{ \left[ 1 + \frac{\beta(1 - \beta)}{2} \right] p\tau + \frac{(p \vartheta)^2}{2c} \right\} + s - \min[p \vartheta, c] + \frac{1 - \beta}{2} \frac{pt(1 - \pi)}{1 - \pi(1 - ft)}. \quad (27)
\]
The equilibrium discount is $d^* = \beta \psi_s^* + (1 - \beta) \psi_b^*$ if $\psi_s^* \geq \psi_b^*$. We then compute the threshold values of the tax morale that define participation in this bargaining game $\tilde{s}_1(v,c)$ and $\tilde{s}_2(v)$ and the corresponding values $\tilde{\nu}_1$ and $\tilde{\nu}_2$ similarly to what we did for the model in complete information, with the only difference that now we have to consider the expected profit. Following the same steps of the integration to find total tax evasion we obtain the following expression:

$$E = \frac{(2 - \beta)(1 - \beta)}{8} \frac{(1 + \vartheta)[1 - (1 - \pi)(1 - ft)]}{2\bar{c}\bar{s}\bar{v}} \left\{ \bar{c}B^2 - \frac{(p\vartheta)^2}{(2 - \beta)(1 + \vartheta)} \left[ B - \frac{1}{3} \frac{p\vartheta}{(2 - \beta)(1 + \vartheta)} \right] \right\}$$ (28)

where

$$B = \frac{pt(1 - \pi)}{1 - \pi(1 - ft)} - \frac{p\tau}{(1 + \vartheta)} - \frac{p\vartheta}{1 + \vartheta} \left[ \frac{1}{2 - \beta} - \frac{p\vartheta}{2\bar{c}} \right].$$ (29)

To compare the expression for tax evasion in the incomplete information model with the one in the complete information benchmark we compute the integral (8) assuming uniform distributions and obtain the following expression:

$$E = \frac{(1 + \vartheta)}{2\bar{c}\bar{s}\bar{v}} \left\{ \bar{c}A^2 - \frac{(p\vartheta)^2}{(1 + \vartheta)} \left[ A - \frac{1}{3} \frac{p\vartheta}{(1 + \vartheta)} \right] \right\}$$ (30)

where

$$A = pt(1 - \pi (1 + f)) - \frac{p\tau}{(1 + \vartheta)}.$$ (31)

Therefore the solution of the model under incomplete information has a very similar structure to the solution of the model under complete information except for the bargaining power that plays no role in the complete information benchmark.

References


Figure 1: Effect of Taxes

Notes: Left panel: tax evasion in percentage terms. Right panel: total government revenue minus the cost of payment instruments scaled to be equal to 100 if equal to the total government revenue in the baseline model specification \((t = 0.3, \tau = 0 \text{ and } \vartheta = 0)\) for each tax evasion level.
Figure 2: Effect of Tax Rebate

Notes: Left panel: tax evasion in percentage terms. Right panel: total government revenue minus the cost of payment instruments for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ($t = 0.3, \tau = 0$ and $\vartheta = 0$) for each tax evasion level.
Figure 3: Effect of Cash Tax

Notes: Left panel: tax evasion in percentage terms. Right panel: total government revenue minus the cost of payment instruments $c$ for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification ($t = 0.3$, $\tau = 0$ and $\vartheta = 0$) for each tax evasion level.
Figure 4: Effect of Tax Rebates and Cash Tax

Notes: Left panel: tax evasion in percentage terms. Right panel: total government revenue minus the cost of payment instruments \( c \) for all the buyers that do not cooperate with tax evaders, scaled to be equal to 100 if equal to the total government revenue in the baseline model specification \( (t = 0.3, \tau = 0, \vartheta = 0) \). Tax rate is fixed to 30%.